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POSSIBLE SIMILARITY SOLUTIONS FOR FREE CONVECTION BOUNDARY LAYERS ADJACENT TO FLAT PLATES IN POROUS MEDIA

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Abstract—The necessary and sufficient conditions under which similarity solutions exist for free convection boundary layers adjacent to flat plates in porous media are examined in this paper. For steady free convection it was determined that similarity solutions exist for vertical plates when the temperature difference between the wall and the environment varies according to power-law and exponential forms and for horizontal plates according to power-law forms. Also, several very specific solutions exist for unsteady free convection about flat plates in a porous medium. For a stable thermally stratified environment, similarity solutions exist only for steady free convection about vertical plates.

NOMENCLATURE

a, wall-to-environment temperature difference function defined in equation (19);
A, arbitrary constant defined in equation (39);
b, similarity transformation function for independent variable, defined in equation (16);
B, arbitrary constant in equation (79b);
c, similarity transformation function for stream function, defined in equation (17);
$C_1, \ldots, C_{10}$, arbitrary constants defined in equations (22)–(26);
C_{11}, environmental stratification arbitrary parameter defined in equation (37b);
$C_{12}, \ldots, C_{33}$, arbitrary constants of integration;
C_p, specific heat;
D_1, D_2, arbitrary constants defined in equations (54) and (75);
E, energy of convected fluid in the boundary layer;
f, similarity stream function variable;
g, acceleration of gravity;
K, permeability of porous material;
k_f, k_m, k_s, thermal conductivity of the fluid, composite material, and porous material, respectively;
L, arbitrary plate length;
m, power-law exponent defined in equation (45);
n, power-law exponent defined in equation (70);
p, power-law exponent defined in equation (80);
P, pressure;
q, local heat-transfer rate;
t, normalized time;
T, temperature;
u, Darcy's velocity in X direction;
v, Darcy's velocity in Y direction;
$\mathbf{V}$, vector velocity;
x, normalized coordinate parallel to surface;
X, dimensional coordinate parallel to surface;
y, normalized coordinate perpendicular to surface;
Y, dimensional coordinate perpendicular to surface.

Greek symbols

α, thermal diffusivity-ratio of composite material conductivity to fluid heat capacity;
β, thermal coefficient of expansion of fluid;
γ, angle of inclined plane measured from horizontal;
δ, normalized boundary-layer thickness;
ε, constant defined in equation (41);
η, independent similarity variable;
θ, temperature similarity variable;
Θ, normalized temperature;
λ, constant defined in equation (66b);
μ, viscosity of fluid;
ρ, density;
σ, ratio of composite material heat capacity to convective fluid heat capacity;
τ, dimensional time;
ϕ, porous material porosity;
ψ, normalized stream function;
Ψ, dimensional stream function.

Subscripts

f, refers to convective fluid;
o, denotes reference quantity;
s, refers to solid porous material;
x, y, t, denote partial differentiation with respect to normalized independent variables;

X, Y, τ, denote partial differentiation with respect to dimensional independent variables;
w, refers to wall;
δ, refers to boundary-layer edge;
∞, denotes conditions at infinity.

1. INTRODUCTION

THERE has recently been considerable interest in the study of free convection in porous media because of the present and potential use of geothermal energy for power production. Most of the earlier theoretical studies are devoted either to the study of the onset of free convection by linear stability analysis [1–3] or to the study of temperature and velocity distributions in enclosures heated from below by numerical methods [4–9]. Recently, a number of similarity solutions have been obtained by Cheng and his co-workers [10–13] for steady free convection about impermeable surfaces embedded in a porous medium with no ambient temperature stratification; in all of these analyses, boundary-layer approximations are employed and power form variations of wall temperature distribution are assumed. Comparison of theory and experimental data shows that the boundary layer approximations in porous layers are valid for moderate to high Rayleigh numbers [14].

It is the purpose of this paper to apply a formalized approach to obtain all possible similarity solutions for free convection boundary layers in a porous medium adjacent to flat plates, including unsteady cases and those with ambient thermal stratification. The solutions presented were obtained not by assuming specific wall-to-ambient temperature distribution forms, but rather by deriving the solution forms from similarity conditions. This approach not only results in more general solution forms than those obtained by Cheng and his co-workers, but results in all other possible solution forms.

The method employed in the present paper is similar to that used by Yang [15], Cheesewright [16], and Gebhart [17] for convective boundary layers in viscous fluids. It consists of defining a new independent variable and two dependent similarity variables in terms of generalized transformation variables. When these similarity variables are substituted into the boundary-layer equations, they are reduced to a set of ordinary differential equations only if the coefficients of the similarity variables and their derivatives can be made constant. These coefficients when made constant are referred to as similarity conditions; they usually contain derivatives of the transformation variables and are thus differential equations themselves. Any solution to the similarity conditions which will result in explicit forms for the transformation variables, will transform the original set of equations into a set of ordinary differential equations, and these are called similarity equations. After each solution is obtained, it is examined for physical feasibility, including whether or not it leads to an unstable thermally stratified environment. In view of the extraordinary length of the paper, numerical results for physically feasible solutions will be presented in subsequent papers.

2. GOVERNING EQUATIONS

To simplify the mathematical analysis, the following assumptions will now be made: (1) the flow is sufficiently slow such that the convective fluid and porous medium are in local thermodynamic equilibrium, (2) the fluid and the porous medium are homogeneous and isotropic, (3) the pressure and temperature are such that the fluid remains in the liquid phase, (4) the porous medium is fully saturated, (5) Darcy's law is valid in thermally active systems, (6) the Boussinesq approximation is valid, i.e. the fluid density can be taken to be constant except in the buoyancy force term, and (7) other fluid and porous medium properties are constant. With these assumptions the equation for the conservation of mass is

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

where the Boussinesq approximation has removed the density term in the equation. The vector $\mathbf{V}$ is the mean velocity flux, or Darcy velocity, and is smaller than the actual mean velocity in the interstices of the porous medium by a factor ϕ, the medium porosity, which has a value less than one.

The equation for the conservation of energy can be written as

$$(\rho C_p)_m T_\tau + (\rho_0 C_p)_f \mathbf{V} \cdot \nabla T = k_m \nabla^2 T, \tag{2a}$$

or in the equivalent form

$$\sigma T_\tau + \mathbf{V} \cdot \nabla T = \alpha \nabla^2 T, \tag{2b}$$

where τ denotes the time; ρ_0 denotes fluid density at a reference temperature T_0; $(\rho C_p)_m$ and k_m are the composite material heat capacity and thermal conductivity defined by $(\rho C_p)_m = (1-\phi)(\rho C_p)_s + \phi(\rho_0 C_p)_f$ and $k_m = (1-\phi)k_s + \phi k_f$ with the subscripts "s" and "f" referring to the porous medium and the fluid, respectively; σ and α are defined by $\sigma = (\rho C_p)_m/(\rho_0 C_p)_f$ and $\alpha = k_m/(\rho_0 C_p)_f$.

The Darcy's law can be written as

$$\frac{\mu}{K}\mathbf{V} = \rho_f g - \nabla P, \tag{3}$$

where g is the gravitational acceleration, K is the permeability of the medium, μ is the viscosity of the fluid, and P is the fluid pressure.

If the density of the fluid can be assumed to be a linearly varying function of temperature, the equation of state can be written as

$$\rho_f = \rho_0[1-\beta(T-T_0)], \tag{4}$$

where β is the thermal coefficient of expansion of the fluid. Equations (1)–(4) are the governing equations for free convection in a porous medium.

3. ANALYSES

Consider a flat impermeable plate, with an inclined angle γ, embedded in a porous medium as shown in Fig. 1 where X and Y are the coordinates parallel and

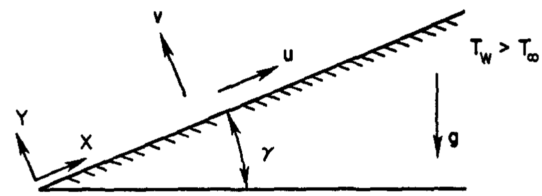

FIG. 1. Coordinate system.

perpendicular to the plate. We will specify that the surface temperature T_w is everywhere greater than, or at least equal to, the surrounding temperature T_∞. This specification is not restrictive since the results for the case where $T_w \leqslant T_\infty$ can be obtained from the results obtained in this paper by a simple change of coordinate. The specification is made to aid in determining physically realistic solutions as will be explained later.

We now introduce the stream function Ψ, defined as

$$u = \Psi_Y, \qquad v = -\Psi_X, \tag{5}$$

where u and v are the Darcy's velocities in the X and Y direction, such that the continuity equation is satisfied. In terms of the stream function and temperature equations (2) and (3) can be written as

$$\Psi_{XX} + \Psi_{YY} = \frac{K\rho_0 \beta g}{\mu}(T_Y \sin\gamma - T_X \cos\gamma), \tag{6}$$

$$\sigma T_\tau + \Psi_Y T_X - \Psi_X T_Y = \alpha(T_{XX} + T_{YY}). \tag{7}$$

It is convenient to express equations (6) and (7) in dimensionless form, and for this purpose, we define the following dimensionless quantities:

$$\psi = \Psi/\alpha, \quad \Theta = \frac{\rho_0 \beta K g L}{\mu\alpha}(T - T_0), \tag{8a,b}$$

$$t = \frac{\tau\alpha}{\sigma L^2}, \quad x = X/L, \quad y = Y/L, \tag{9a,b,c}$$

where L is an arbitrary length. In terms of the dimensionless variables, equations (6) and (7) can be written as

$$\psi_{xx} + \psi_{yy} = \Theta_y \sin\gamma - \Theta_x \cos\gamma, \tag{10}$$

$$\Theta_t + \psi_y \Theta_x - \psi_x \Theta_y = \Theta_{xx} + \Theta_{yy}. \tag{11}$$

We now assume that convection takes place in a thin layer adjacent to the heated surface so that the change in physical quantities, with respect to x, are small compared to those with respect to y. From an order of magnitude estimate it can be shown that equations (10) and (11) can be approximated by

$$\psi_{yy} = \Theta_y \sin\gamma - \Theta_x \cos\gamma, \tag{12}$$

$$\Theta_t + \psi_y \Theta_x - \psi_x \Theta_y = \Theta_{yy}. \tag{13}$$

If an impermeable surface with prescribed temperature is embedded in the porous medium, the boundary conditions at the wall are

$$y = 0, \quad \psi_x = 0 \quad \text{and} \quad \Theta = \Theta_w. \tag{14}$$

At a great distance from the wall, i.e. $y \to \infty$, the velocity parallel to the plate is not affected by the heated surface. Thus, we have

$$y \to \infty, \quad \psi_y = 0 \quad \text{and} \quad \Theta = \Theta_\infty, \tag{15a,b}$$

where equation (15a) implies that any variation in Θ_∞ does not induce fluid motion, i.e. a stable environment is assumed. As we shall see, the nature of the similarity transformation of the independent variable does not allow the boundary conditions at $x = 0$ or the initial conditions to be independently prescribed.

Similarity conditions

We now examine the necessary conditions for which similarity solutions exist. To this end we now define a new independent similarity variable

$$\eta = yb(x, t), \tag{16}$$

and two new dependent similarity variables

$$f(\eta) = \psi(x, y, t)/c(x, t), \tag{17}$$

and

$$\theta(\eta) = [\Theta(x, y, t) - \Theta_\infty(x, t)]/a(x, t), \tag{18}$$

where

$$a(x, t) = \Theta_w(x, t) - \Theta_\infty(x, t), \tag{19}$$

and b and c are the transformation variables to be determined.

By using equations (16)–(19), equations (12) and (13) can be written in the following form:

$$f'' + (C_1\eta - C_2)\theta' + C_3\theta + C_4 = 0, \tag{20}$$

$$\theta'' - C_5 f'\theta - C_6 f' + C_7 f\theta' - C_8\eta\theta' - C_9\theta - C_{10} = 0, \tag{21}$$

where

$$C_1 = \frac{ab_x}{b^3 c}\cos\gamma, \quad C_2 = \frac{a}{bc}\sin\gamma, \tag{22a,b}$$

$$C_3 = \frac{a_x}{b^2 c}\cos\gamma, \quad C_4 = \frac{\Theta_{\infty x}}{b^2 c}\cos\gamma, \tag{23a,b}$$

$$C_5 = \frac{ca_x}{ab}, \quad C_6 = \frac{c\Theta_{\infty x}}{ab}, \tag{24a,b}$$

$$C_7 = \frac{c_x}{b}, \quad C_8 = \frac{b_t}{b_3}, \tag{25a,b}$$

$$C_9 = \frac{a_t}{ab^2}, \quad C_{10} = \frac{\Theta_{\infty t}}{ab^2}. \tag{26a,b}$$

We now observe that equations (20) and (21) become ordinary differential equations only if the quantities $C_1, C_2, \ldots, C_{10}$, as given by equations (22)–(26) can be made constants with respect to x and t. The requirements that these quantities are constant will lead to explicit expressions for a, b, c, and Θ_∞.

In terms of the similarity variables, the boundary conditions, equations (14) and (15), become

$$\eta = 0, \quad f = 0 \quad \text{and} \quad \theta = 1, \tag{27a,b}$$

$$\eta \to \infty, \quad f' = 0 \quad \text{and} \quad \theta = 0. \tag{28a,b}$$

It should be noted, as in the case of similarity solutions for viscous fluids [15], that because of the nature of equation (16), specifying the conditions at $y = 0$ and $y \to \infty$ are all that are required to solve equations (20) and (21). This means that we cannot independently specify the initial temperature distribution or boundary conditions at $x = 0$, but must accept whatever distribution and conditions result from the specific solutions obtained. For many solutions, the resulting conditions will match the physical values that may be desired, however, some solutions will not; a specific case where the latter situation occurs will be discussed later.

Heat- and mass-transfer quantities

It is convenient at this point to express a number of heat- and mass-transfer quantities in terms of a, b, and c. The local surface heat flux is given by

$$q_w = -k_m[T_Y]_w = \left(\frac{k_m \alpha \mu}{\rho_0 \beta g K L}\right) ab[-\theta'(0)]/L$$
$$= k_m(T_w - T_\infty)b[-\theta'(0)]/L. \quad (29)$$

This equation shows a relationship between the surface heat flux and the wall-to-ambient temperature difference. Thus, although the similarity solutions given in this paper assume the wall-to-ambient temperature difference is to be prescribed, we could have just as well assumed that the surface heat flux (i.e. temperature gradient at the wall) is to be prescribed; thus the following similarity solutions are applicable to both cases.

The velocity component parallel to the surface in the boundary layer can be written as

$$u = \frac{\alpha bc}{L} f'(\eta), \quad (30)$$

and the energy convected in the boundary layer at any x and t is given by

$$E(x, t) = \rho_0 C_p \int_0^\infty u(T - T_\infty)\,dY$$
$$= \left(\frac{C_p \alpha^2 \mu}{\beta g K L}\right) ac \int_0^\infty \theta f'\,d\eta. \quad (31)$$

Finally, if we arbitrarily define the thermal boundary-layer thickness as the point at $y = \delta$, or $\eta = \eta_\delta$, where $\theta = 0.01$, we have from equation (16)

$$\delta = \eta_\delta / b. \quad (32)$$

Feasibility considerations

To examine the feasibility of the solutions, several factors must be considered. The first concerns whether the solution describes physical realizable flows. For the case where $T_w \geq T_\infty$, it can be assumed that the velocity parallel to the plate u, the boundary-layer thickness δ, and the total energy convected E, will increase or at least be constant with respect to x [17]. These conditions must also hold for transient cases where the boundary layer is building up with time. It follows from equations (30), (31), and (32) that:

(i) bc must be constant or increasing with x or t;
(ii) b must be constant or decreasing with x or t; (33)
(iii) ac must be constant or increasing with x or t.*

*An exception to condition (33) occurs for the case of the exponential solution where δ (which is not zero at $x = 0$) can decrease with increasing u and E [20].

It should be noted that the opposite of the above conditions will hold for the transient case where the boundary layer is decaying with time (both u and E will be decreasing with time).

A second factor affecting feasibility is whether the resulting similarity equations can satisfy the boundary conditions at infinity. It is worth noting that higher derivatives of θ and f' must approach zero outside the boundary layer as is evident from physical considerations. Applying these conditions at infinity to equations (20) and (21), we conclude that

$$C_4 = C_{10} = 0. \quad (34a,b)$$

Equation (34b) together with equation (26b) implies that $\Theta_{\infty,t} = 0$, i.e. Θ_∞ must in all cases be independent of t. For an inclined or a horizontal plate, equation (34a) together with equation (23b) implies that Θ_∞ must be independent of x.

A third factor, also related to satisfying boundary conditions, occurs for the solutions where the similarity equations reduce to $f'' = 0$ and/or $\theta'' = 0$. Neither of these equations can truly satisfy equations (27) and (28) and result in flow with boundary-layer behavior. Thus any solution resulting in either of these forms for f or θ is considered infeasible, and will not be presented in the following as a possible solution.

For free convection about vertical or inclined surfaces, a fourth factor affecting feasibility concerns whether the solutions result in a stable environment. Within the framework of the Boussinesq approximation, a stable environment requires that the density must be constant or decreasing with increasing x ($\rho_{\infty,x} \leq 0$). This implies that the surrounding temperature must be increasing with x (i.e. $\theta_{\infty,x} \geq 0$) as is evident from equation (4). It follows from equation (24b) and the fact that a and b are positive (it can be shown that c must also be positive) that

$$C_6 \geq 0. \quad (35)$$

Equation (35) is a sufficient condition that ensures a stable stratified environment for which no motion exists outside the thermal boundary layer. In reality since density also depends on pressure which decreases upward, the necessary condition for a stable environment is that the actual density stratification be greater in absolute magnitude than the "adiabatic" one [17, 18].

Particular solutions

In this section the possible similarity solutions for free convection about a plane surface in a porous medium will be derived. As is evident from a thorough study of equations (22)–(26), there are no explicit expressions for a, b, c, and Θ_∞ which result in non-zero values for all of the eight arbitrary constants C_i, where $i = 1, 2, 3, 5, \ldots, 9$. In other words, since there are only the four unknowns, a, b, c, and Θ_∞, the general similarity conditions result in more equations than unknowns and therefore are not all independent equations. The approach then is to examine particularized cases (e.g. steady flow over vertical plates) where some of the C_i are zero and where the remaining similarity conditions contain no more than four independent equations.

For any particular problem we must choose arbitrary non-zero values for four of the C_i's involved in a solution. Any other of the eight constants involved in a particular solution will not be arbitrary but will be related to those assigned a value. For example, from equations (22a), (23a), (24a), and (25a), one relation among four of the constants that is used in the following analysis is

$$\frac{C_1}{C_3} = 1 - \frac{2C_7}{C_5}. \tag{36}$$

However, it is our purpose in this paper to leave the solution in a general form in order to enhance the range of applicability. Thus we will assign values to the C_i only when generality in the forms of the unknowns is not affected. Where practical we will use constants of integration to maintain generality, as we desire to assign values to as many of the C_i as possible in order to make solutions of the similarity equations as general as possible. In many cases we have defined new constants related to the C_i's in order to obtain a simple form for a, the prescribed temperature difference between the wall and the environment.

Because of the similarity between equations (24a) and (24b), we find that expressions for Θ_∞ will depend on whether a_x is zero or non-zero.

(A) For $a_x \neq 0$, we have from equations (24a) and (24b)

$$\Theta_{\infty x} = C_{11} a_x, \tag{37a}$$

where

$$C_{11} = C_6/C_5. \tag{37b}$$

Integration of equation (37a) gives

$$\Theta_\infty = C_{11} a + C_{12}, \tag{38a}$$

where C_{12} is a constant of integration. Combining equations (19) and (38a) we have

$$\Theta_w = (1+C_{11})a + C_{12}. \tag{38b}$$

We note that C_{11} can be taken as an arbitrary constant parameter, instead of C_6, if desired. It follows from equation (38a) and (38b) that $C_{11} = 0$ results in the case of isothermal surroundings, and $C_{11} = -1$ corresponds to the isothermal wall solution.

(B) For $a_x = 0$, i.e. $a = A$ = constant, we have from equation (19)

$$\Theta_w = A + \Theta_\infty, \tag{39}$$

and it is found that Θ_∞ can take on two different forms.

(i) When b and c are not constant, we can eliminate b from equations (24b) and (25a) to obtain $\Theta_{\infty x}$ in terms of c, which can then be integrated to give

$$\Theta_\infty = \frac{C_6}{C_7} A \ln c + C_{12}. \tag{40a}$$

(ii) When b and c are also constant, we can integrate equation (24b) to give

$$\Theta_\infty = \frac{Ab}{c} C_6 x + C_{12}. \tag{40b}$$

Since the form of Θ_∞ depends entirely on the form of a, b, and c, it is not necessary to consider the similarity conditions involving Θ_∞ further. This means that in maintaining generality in the form for the unknowns, we can arbitrarily assign values for up to three, instead of four, of the constants in a solution.

In the remainder of this paper, we will present the possible solutions to equations (22)–(26). We will consider the steady state solutions first, followed by the unsteady solutions.

Case 1. *Steady free convection about vertical flat plates*

For this case ($\gamma = \pi/2$), we have $C_1 = C_3 = C_4 = C_8 = C_9 = C_{10} = 0$. We now solve for a, b, and c taking the remaining quantities C_2, C_5, and C_7 as constants. Equations (22b), (24a), and (25a) can then be combined to give an equation in terms of c, which when integrated gives

$$c^{(\varepsilon-1)/\varepsilon} c_x = C_{13}, \tag{41}$$

where C_{13} is a constant of integration and $\varepsilon = C_7/C_5$. The form of the integral of equation (41) depends on whether $\varepsilon \neq 1/2$ or $\varepsilon = 1/2$.

(A) *Power-law variation with* x. For $\varepsilon \neq 1/2$, equation (41) can be integrated to give

$$c = (C_{15}x + C_{14})^{\varepsilon/(2\varepsilon-1)}, \tag{42}$$

where we have defined $C_{15} = [(2\varepsilon-1)/\varepsilon]C_{13}$. From equations (42), (25a), and (22b), we can obtain

$$b = \frac{C_{15}}{C_7}\left(\frac{\varepsilon}{2\varepsilon-1}\right)[C_{15}x + C_{14}]^{(1-\varepsilon)/(2\varepsilon-1)}, \tag{43}$$

$$a = \frac{C_2 C_{15}}{C_7}\left(\frac{\varepsilon}{2\varepsilon-1}\right)[C_{15}x + C_{14}]^{1/(2\varepsilon-1)}. \tag{44}$$

In order to simplify and yet maintain a general form for a, we define a new constant m, as $m = 1/(2\varepsilon-1)$, and arbitrarily select $C_2 = 1$, and $C_7 = \varepsilon/(2\varepsilon-1)$. The expressions for a, b, and c then become

$$a = C_{15}(C_{15}x + C_{14})^m, \tag{45}$$

$$b = C_{15}(C_{15}x + C_{14})^{(m-1)/2}, \tag{46}$$

$$c = (C_{15}x + C_{14})^{(m+1)/2}. \tag{47}$$

Taking Θ_∞ from equation (38a), equations (24a) and (24b) show that

$$C_5 = m \quad \text{and} \quad C_6 = C_{11} m. \tag{48a,b}$$

With these values for C_2, C_5, C_6, and C_7, equations (20) and (21) become

$$f'' - \theta' = 0, \tag{49}$$

$$\theta'' + \frac{m+1}{2} f\theta' - mf'(\theta + C_{11}) = 0. \tag{50}$$

It should be noted that equations (45)–(47) are also valid in the special case where $m = 0$, which corresponds to a constant temperature difference solution with $a = A$ = constant. This case allows Θ_∞ to take on the logarithmic form given by equation (40a),

$$\Theta_\infty = C_6 A \ln(C_{15}x + C_{14}) + C_{12}, \tag{51}$$

where we have arbitrarily assigned $C_7 = 1/2$ and Θ_w is given by equation (39). We also have from equation (48a) that $C_5 = 0$, and consequently, equations (20) and (21) are given by

$$f'' - \theta' = 0, \tag{52}$$

$$\theta'' + \tfrac{1}{2} f\theta' - C_6 f' = 0. \tag{53}$$

Consideration of feasibility for the $\varepsilon \neq 1/2$ case leads to restrictions on the value of m. Conditions (33) show that δ will be constant or increase with x if $m \leqslant 1$ and that the same is true for u if $m \geqslant 0$. The convected energy will be increasing with x if $m > -1/3$. Thus, we conclude that the present case is physically realizable for m in the range $0 \leqslant m \leqslant 1$. If equations (45) and (46) are substituted in equation (29), we find that $m = 1/3$ corresponds to the constant surface heat flux solution.

Equations (48b) and (35) show that mC_{11} must be positive for a thermally stable environment. Since m is restricted to be positive, we find that $C_{11} \geqslant 0$. This expression rules out the possible similarity solution for an isothermal vertical wall in a thermally stratified environment ($C_{11} = -1$).

We note from equations (30), (32) and (45)–(47) that if $C_{14} \neq 0$, then u and δ will not vanish at $x = 0$, meaning that some plate length and temperature difference must precede $x = 0$ in order for the boundary-layer thickness to build up. Thus, one effect of C_{14} is merely to allow a shift in the x-coordinate. If we assume that $x = 0$ corresponds to the leading edge of the plate, we have $C_{14} = 0$, and a simplified general form for a, b, and c can be written as

$$a = D_1 x^m, \quad b = (D_1 x^{m-1})^{1/2}, \quad c = (D_1 x^{m+1})^{1/2}, \tag{54a,b,c}$$

where we have defined $D_1 = C_{15}^{m+1}$. For the special case of $C_{11} = 0$ (which corresponds to a vertical flat plate with power-law variation of wall temperature distribution in an isothermal surrounding), numerical solutions to equations (49) and (50) with equation (54) have been obtained by Cheng and Minkowycz [10].

(B) *Exponential variation with* x. For the case where $\varepsilon = 1/2$, we can integrate equation (41), giving

$$c = C_{16} \exp(C_{13} x), \tag{55}$$

where C_{16} is a constant of integration. Then b and c from equations (25a) and (22b) become

$$b = \frac{C_{16}}{C_7} C_{13} \exp(C_{13} x), \tag{56}$$

$$a = \frac{C_2 C_{16}^2 C_{13}}{C_7} \exp(2C_{13} x). \tag{57}$$

To simplify the expression for a, yet maintain generality, we define $C_{17} = 2C_{13}$, $C_{18} = C_{16}^2 C_{17}$, and arbitrarily let $C_2 = 1$, and $C_7 = 1/2$. Equations (57), (56), and (55) then become

$$a = C_{18} \exp(C_{17} x), \tag{58}$$

$$b = (C_{18} C_{17})^{1/2} \exp(C_{17} x/2), \tag{59}$$

$$c = (C_{18}/C_{17})^{1/2} \exp(C_{17} x/2). \tag{60}$$

Also we find from equation (24a) that $C_5 = 1$, and using equations (37b) and (38a) the similarity equations given by equations (20) and (21) for this case are

$$f'' - \theta' = 0, \tag{61}$$

$$\theta'' + \tfrac{1}{2} f\theta' - f'(\theta + C_{11}) = 0. \tag{62}$$

Equation (59) shows that at no finite value of x does the boundary-layer thickness go to zero. The same situation occurs with the exponential form for free convection about a vertical flat plate in a viscous fluid and is discussed by Gebhart and Mollendorf [19]. Their conclusion is that the solution is meaningful if the momentum and energy entering at the leading edge is small compared to that exiting at L, where L is the length of plate of interest. Mathematically, this requires that C_{17} is large compared to unity. This problem arises in connection with not being able to specify boundary conditions at $x = 0$ due to the nature of the y to η transformation, as previously mentioned. The exponential solution appears physically realistic even though u and E are increasing with x while δ decreases with x.

(C) *Solution for constant* a *and* b. If a and b are both constant, then equation (22b) shows that c must be constant also. The only variation allowed is in Θ_w, which is given by equation (40b). We can simplify equation (40b) by letting $b = 1$ and $c = A$, obtaining

$$\Theta_w = C_6 x + C_{12}. \tag{63}$$

The similarity equations for the present problem are

$$f'' - \theta' = 0, \tag{64}$$

$$\theta'' - C_6 f' = 0. \tag{65}$$

This solution, although physically realistic, applies only to the special situation where δ, u, and E are constant with x. We note that equations (64) and (65) are readily solvable in closed form.

Case 2. *Steady free convection about inclined planes*

For a plane inclined at an angle γ with respect to the horizontal, we have $C_4 = C_8 = C_9 = C_{10} = 0$. Thus similarity solutions exist if C_1, C_2, C_3, C_5, and C_7 can be made constant. As in Case 1, we first assume that C_2, C_5, and C_7 are constant which leads to an ordinary differential equation for c given by equation (41). Further integration of equation (41) depends on whether $\varepsilon \neq 1/2$ or $\varepsilon = 1/2$. It can be shown (see [20] for details) that, for the former case, the solution is not physically realizable as both u and E are found to decrease with x, while for the latter case it is not possible to make C_3 independent of x.

Although no exact feasible similarity solutions for free convection about inclined plates in a porous medium were found, approximate solutions exist if we make the assumption that the effect of the normal component of gravity can be neglected, i.e. $g(\cos\gamma)T_X \ll g(\sin\gamma)T_Y$. In this case the solutions for an inclined plane will be identical to those for a vertical plate if g is replaced by $g\sin\gamma$.

Case 3. Steady free convection about horizontal plates

For this case (where $\gamma = 0$), we have $C_2 = C_4 = C_8 = C_9 = C_{10} = 0$. Since Θ_∞ must be independent of x for a horizontal plate as discussed earlier, we have therefore $C_6 = C_{11} = 0$. Similarity solutions exist for this case if we can find a, b, and c such that C_1, C_3, C_5, and C_7 become constant. We can make use of equations (22a), (23a), (24a) and (25a) to obtain an equation for c, which can be integrated once to give

$$c^{-\lambda}c_x = C_{19}, \tag{66a}$$

where C_{19} is a constant of integration and

$$\lambda = C_1C_5/(C_3C_7). \tag{66b}$$

Further integration of equation (66a) leads to two different forms depending on the value of λ as follows.

(A) *Power-law variation with* x. For $\lambda \neq 1$, equation (66a) can be integrated to give

$$c = (C_{21}x + C_{20})^{1/(1-\lambda)}, \tag{67}$$

where we have defined $C_{21} = (1-\lambda)C_{19}$. Using equations (67), (25a), and (23a), we find

$$b = \frac{C_{21}}{C_7(1-\lambda)}[C_{21}x + C_{20}]^{\lambda/(1-\lambda)}, \tag{68}$$

$$a = \frac{C_3C_{21}}{C_5C_7(1-\lambda)}[C_{21}x + C_{20}]^{(2+\lambda)/(1-\lambda)}. \tag{69}$$

Maintaining a simple general form for a, we define $n = (2+\lambda)/(1-\lambda)$, and arbitrarily set $C_7 = 1/(1-\lambda)$ and $C_3 = n$. Equations (69), (68), and (67) then become

$$a = C_{21}(C_{21}x + C_{20})^n, \tag{70}$$

$$b = C_{21}(C_{21}x + C_{20})^{(n-2)/3}, \tag{71}$$

$$c = (C_{21}x + C_{20})^{(n+1)/3}. \tag{72}$$

With the above values for C_3 and C_7, we can show that $C_5 = n$ and $C_1 = (n-2)/3$, in which case equations (20) and (21) become

$$f'' + \left(\frac{n-2}{3}\right)\eta\theta' + n\theta = 0, \tag{73}$$

$$\theta'' - nf'\theta + \frac{n+1}{3}f\theta' = 0. \tag{74}$$

To determine the feasible range of n, we first consider the boundary-layer thickness, which is constant or increasing with x if $n \leqslant 2$. The requirements that u and E be constant or increasing with x gives $n \geqslant 1/2$ and $n \geqslant -1/4$ respectively. Thus we conclude that n must be restricted to the range $1/2 \leqslant n \leqslant 2$. It is worth noting that a constant surface heat flux solution for a horizontal flat plate occurs when $n = 1/2$. Furthermore, if we set $C_{20} = 0$ in equations (70)–(72), which amounts to a shift of the coordinate system in the x-direction when the boundary layer does not begin at $x = 0$, equations (70)–(72) can be rewritten as

$$a = D_2x^n, \quad b = (D_2x^{n-2})^{1/3}, \quad c = (D_2x^{n+1})^{1/3}, \tag{75a,b,c}$$

where we have defined $D_2 = C_{21}^{n+1}$. Note that Θ_w and Θ_∞ are given by equations (38) with $C_{11} = 0$. The numerical solutions to equations (73) and (74) have been obtained by Cheng and Chang [12].

(B) *Exponential variation with* x. Proceeding as in the previous case, for $\lambda = 1$ we can obtain c from the integration of equation (66a) (using C_{19} as a constant of integration) and obtain b and a from equations (25a), (23a) and (24a), with the result

$$a = C_{24}\exp(C_{23}x), \tag{76a}$$

$$b = (C_{24}C_{23})^{1/2}\exp(C_{23}x/3), \tag{76b}$$

$$c = (C_{24}/C_{23})^{1/2}\exp(C_{23}x/3), \tag{76c}$$

where C_{22} is another constant of integration, and where we have defined $C_{23} = 3C_{19}$, $C_{24} = C_{22}^2C_{23}$. To simplify the expression for a in equation (76), we have arbitrarily set $C_1 = C_7 = 1/3$, and thus $C_3 = C_5 = 1$. With these values, equations (20) and (21) become

$$f'' + \tfrac{1}{3}\eta\theta' + \theta = 0, \tag{77}$$

$$\theta'' - f'\theta + \tfrac{1}{3}f\theta' = 0. \tag{78}$$

The discussion in Case 1-B also holds for this solution, and we conclude that the solution is meaningful if the value of C_{23} is large compared to unity.

Case 4. Asymptotic solutions for unsteady free convection about inclined planes

The first set of unsteady solutions considered here are ones where there is no variation of any variable in x. These cases are referred to as asymptotic solutions, and are valid only at sufficiently large distances from the leading edge such that changes in the variables with x can be considered negligible. When all derivatives with respect to x are equal to zero, we find that $C_1 = C_3 = C_4 = C_5 = C_6 = C_7 = C_{10} = 0$. Thus we only need to find a, b, and c such that C_2, C_8, and C_9 become constant. From equation (25b), it is apparent that C_8 will be a constant when

$$b = (C_{25} - 2C_8t)^{-1/2}, \tag{79a}$$

or if

$$b = B = \text{constant}, \tag{79b}$$

where C_{25} is a constant of integration.

(A) *Power-law variation with time.* With b given by equation (79a), we can obtain a and c from equations (26a) and (22b) to give

$$a = C_{26}(C_{25} - 2C_8t)^p, \tag{80}$$

$$c = C_{26}(C_{25} - 2C_8t)^{p+1/2}, \tag{81}$$

where C_{26} is another constant of integration, $p = -C_9/2C_8$, and we have arbitrarily set $C_2 = \sin\gamma$. It follows that the similarity equations become

$$f'' - (\sin\gamma)\theta' = 0, \tag{82}$$

$$\theta'' - C_8[\eta\theta' - 2p\theta] = 0. \tag{83}$$

Equation (80) shows that the constant C_{26} should always be positive to maintain $\Theta_w \geqslant \Theta_\infty$, and the quantity $(C_{25} - 2C_8t)$ should always be positive for any value of t. The wall and surrounding temperatures can be made to increase or decrease with time by proper choice of the constants C_{25}, C_8, or p. For the case of a transient solution with boundary-layer thickness increasing with time and zero initial thickness, we find from equations (32) and (79a) that C_{25} must vanish and $C_8 < 0$. We find also that we must have $p \geqslant 0$ if u is to increase or be constant with time. For the case when the boundary-layer thickness is finite at $t = 0$ and starts to decay for $t > 0$, the value of C_{25} will have a non-zero value and we again find $p \geqslant 0$ for decreasing or constant u. Examination of equations (29), (79a), and (80) shows that a constant wall heat flux solution exists for $p = 1/2$.

(B) *Exponential variation with time.* For $b = B$ = constant we have $C_8 = 0$, and we can obtain a and c from equations (26a) and (22b) to give

$$a = C_{27}\exp(B^2 t), \tag{84}$$

$$c = \frac{C_{27}}{B}\exp(B^2 t), \tag{85}$$

where C_{27} is a constant of integration and we have arbitrarily set $C_2 = \sin\gamma$ and $C_9 = 1$. With these values we have the following similarity equations,

$$f'' - (\sin\gamma)\theta' = 0, \tag{86}$$

$$\theta'' - \theta = 0, \tag{87}$$

which we note can be solved in closed form. C_{27} must be positive, consequently velocity and convected energy must be increasing with time. The boundary-layer thickness, however, will remain constant.

Case 5. Unsteady free convection about vertical plates

For this problem (where $\gamma = \pi/2$), we have $C_1 = C_3 = C_4 = C_6 = C_{10} = 0$. If we can find a, b, and c such that C_2, C_5, C_7, C_8, and C_9 are constant, then similarity solutions exist. It is noted that the conditions for which C_2, C_5, and C_7 are constant have been obtained in equations (42)–(44) where C_{14} and C_{15} can now be functions of time. The conditions for which C_2, C_8 and C_9 are constant are given in equations (79)–(81) where C_{25} and C_{26} can now be functions of x. Compatibility between these sets of equations can occur only if the various constants of integration can be defined such that a, b and c from each set of equations can be made equal. Taking b for example, we have from equations (43) and (79a) [we note that the use of equation (79b) does not result in a solution for this case]

$$b = \frac{C_{15}(t)}{C_7}\left(\frac{\varepsilon}{2\varepsilon-1}\right)[C_{15}(t)x + C_{14}(t)]^{(1-\varepsilon)/(2\varepsilon-1)} = [C_{25}(x) - 2C_8 t]^{-1/2}. \tag{88}$$

A careful examination of equation (88) shows that the equality will hold when $\varepsilon = 1$ and $\varepsilon = \infty$, or when $m = 1$ and $m = 0$ since $m = 1/(2\varepsilon - 1)$.

(A) *Product combination of* x *and* t(ε = 1). In this instance equation (88) reveals that C_{25} must be a constant and that

$$C_{15}(t) = C_7[C_{25} - 2C_8 t]^{-1/2}. \tag{89}$$

Using equation (89) and requiring a and c from each set be equal, results in the expressions

$$a = C_2(C_7 x + C_{28})/(C_{25} - 2C_8 t), \tag{90}$$

$$b = (C_{25} - 2C_8 t)^{-1/2},$$
$$c = [C_7 x + C_{28}](C_{25} - 2C_8 t)^{-1/2}, \tag{91a,b}$$

where C_{28} is an arbitrary constant and where we have defined $C_{14}(t) = C_{28}(C_{25} - 2C_8 t)^{-1/2}$ and $C_{26}(x) = C_2(C_7 x + C_{28})$. We note that for these equations, $p = -1$, which is not a feasible value as it leads to conflicting time variations of δ with u and E; we thus conclude that this solution is not physically realizable.

(B) *Linear summation of* x *and* t($\varepsilon = \infty$). An investigation of this case reveals that the only solution possible occurs when a is constant, and from equation (44), we have $a = A = C_2 C_{15}/2C_7$, which can be simplified by arbitrarily defining $C_2 = 1$, and $C_7 = 1/2$. From equation (80) we also have $p = 0$ and $C_{26} = A$, and from equations (24a) and (26a) we obtain $C_5 = C_9 = 0$. By defining $C_{14}(t) = A^2(C_{29} - 2C_8 t)$ and $C_{25}(x) = x/A + C_{29}$ where C_{29} is an arbitrary constant, the expressions for b and c can be shown to become

$$b = \left(\frac{x}{A} + C_{29} - 2C_8 t\right)^{-1/2},$$

and (92a,b)

$$c = A\left(\frac{x}{A} + C_{29} - 2C_8 t\right)^{1/2}.$$

The similarity equations can then be written as

$$f''' - \theta' = 0, \tag{93}$$

$$\theta'' + \tfrac{1}{2}f\theta' - C_8\eta\theta' = 0. \tag{94}$$

Since $p = 0$ and $m = 0$ are within the physically realizable ranges for a vertical plate, we conclude that the solution is feasible. For $C_{29} = 0$ and $C_8 < 0$, the solution will apply to a growing boundary layer with zero thickness initially. If $C_{29} \neq 0$ and $C_8 > 0$, the solution will apply to a decaying boundary layer.

Case 6. Unsteady free convection about horizontal plates

For this problem we have $C_2 = C_4 = C_6 = C_{10} = 0$, and we will try to find the expressions for a, b, and c which satisfy the other similarity conditions. The expressions for a, b, and c such that C_1, C_3, C_5, and C_7 are constant are given by equations (67)–(69), whereas the expressions for a and b such that C_8 and C_9 are constant are given by equations (79a) and (80). As before, requiring the two expressions for b to be equal, we have

$$\frac{C_{21}(t)}{C_7(1-\lambda)}[C_{21}(t)x + C_{20}(t)]^{\lambda/(1-\lambda)} = [C_{25}(x) - 2C_8 t]^{-1/2}. \tag{95}$$

This equality holds for $\lambda = 0$ and $\lambda = -1$ or for $n = 2$ and $n = 1/2$ since $n = (2+\lambda)/(1-\lambda)$.

(A) *Product combination of* x *and* t ($\lambda = 0$). For this case, equation (95) shows that C_{25} must be a constant and

$$C_{21}(t) = [C_{25} - 2C_8 t]^{-1/2} C_7. \quad (96)$$

Using equation (96), requiring a from each set be equal, and obtaining c from equation (25a), we have

$$a = \frac{C_3}{2C_7}(C_{25} - 2C_8 t)^{-3/2}(C_7 x + C_{30})^2, \quad (97)$$

$$b = (C_{25} - 2C_8 t)^{-1/2}, \quad (98)$$

$$c = (C_{25} - 2C_8 t)^{-1/2}(C_7 x + C_{30}), \quad (99)$$

where C_{30} is an arbitrary constant and where we have defined $C_{20}(t) = C_{30}(C_{25} - 2C_8 t)^{-1/2}$ and $C_{26}(x) = (C_3/2C_7)(C_7 x + C_{30})^2$. Comparison of equation (97) with equation (80) shows that $p = -3/2$, which as discussed previously is not a feasible value. We thus conclude that this solution is not physically realizable and will not be discussed further.

(B) *Linear summation of* x *and* t ($\lambda = -1$). This case arises when $C_{20}(t)$ and $C_{25}(x)$ are defined such that the quantities in brackets will cancel. Then $C_{21} = 2C_7$ and we can define

$$C_{20}(t) = -2C_8 t + C_{31}, \quad (100)$$

$$C_{25}(x) = 2C_7 x + C_{31}, \quad (101)$$

where C_{31} is an arbitrary constant. From equations (36) and (66b) we find $C_5 = C_7$ and $C_1 = -C_3$. Requiring that a from each set also be equal and obtaining c from equation (25a), it can be shown that

$$a = \frac{C_3}{C_7}(2C_7 x + C_{31} - 2C_8 t)^{1/2}, \quad (102)$$

$$b = (2C_7 x + C_{31} - 2C_8 t)^{-1/2}, \quad (103)$$

$$c = (2C_7 x + C_{31} - 2C_8 t)^{1/2}. \quad (104)$$

The similarity equations are

$$f'' + C_3(-\eta\theta' + \theta) = 0, \quad (105)$$

$$\theta'' + C_7(f\theta' - f'\theta) - C_8(\eta\theta' - \theta) = 0. \quad (106)$$

We note that for the present problem $n = 1/2$ and $p = 1/2$, which are within the feasible ranges. This solution can be applied to growing or decaying boundary layers depending on the values of C_{31} and C_8. From equation (103), we must require that the quantity $(2C_7 x + C_{31} - 2C_8 t)$ be positive. We note also that since a times b equals a constant, this case is equivalent to a constant wall heat flux solution as is evident from equation (29).

(C) *Linear summation of* x *and* t *for constant temperature difference*. For the horizontal plane there is also a solution for $a = A =$ constant which is not directly obtainable from the previous solutions. We have $C_2 = C_3 = C_4 = C_5 = C_6 = C_9 = C_{10} = 0$, and we must make C_1, C_7, and C_8 constant. The equation for b is again given by equation (79a). Solving c from equation (22a) in terms of b and substituting into equation (25a), we find that C_7 can be constant only when

$$[C_{25}(x)]_{xx} = 0, \quad (107)$$

resulting in $C_7 = 0$. Integrating equation (107) twice yields

$$C_{25}(x) = C_{32}x + C_{33}, \quad (108)$$

where C_{32} and C_{33} are constants of integration. Substituting equation (108) into equation (79a) and (22a), we have

$$a = A, \quad b = (C_{32}x + C_{33} - 2C_8 t)^{-1/2}, \quad c = -AC_{32}/2C_1. \quad (109a,b,c)$$

Applying the feasibility requirements to this solution, we find that it is impossible to make δ and u both increase with x at the same time. We thus conclude that this case is not physically realizable.

Table 1. Physically realizable solutions for flat plates

Case	Flow state	Plate angle	Solution form for $T_w - T_\infty$	Environmental stratification
1-A	steady	vertical	AX^n	yes
1-B	steady	vertical	$A\exp[BX]$	yes
1-C	steady	vertical	A	yes
3-A	steady	horizontal	AX^n	no
3-B	steady	horizontal	$A\exp[BX]$	no
4-A	unsteady	inclined	At^n	no
4-B	unsteady	inclined	$A\exp[B\tau]$	no
5-B	unsteady	vertical	A	no
6-B	unsteady	horizontal	$(AX + B\tau)^{1/2}$	no

4. CONCLUDING REMARKS

A systematic method was used to derive all the possible similarity solutions for free convection in porous layers about flat plates. The solutions obtained in the present paper are more general than those appearing in the previous publications and include the unpublished cases of unsteady state and thermally stratified surroundings. A summary of all physically realizable solutions is presented in Table 1.

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SOLUTIONS POSSIBLES DE SIMILITUDE POUR LES COUCHES LIMITES DE CONVECTION NATURELLE ADJACENTS A DES SURFACES PLANES DE MILIEUX POREUX

Résumé—On examine les conditions nécessaires et suffisantes pour que des solutions de similitude existent concernant les couches limites de convection naturelle sur des surfaces planes de milieux poreux. Pour la convection permanente, on conclut que les solutions de similitude existent pour des plaques verticales quand la différence de température entre la paroi et l'environnement varie suivant des lois de puissance et d'exponentielle et, pour des plaques horizontales, suivant des lois de puissance. Il existe aussi des solutions très particulières pour la convection naturelle non stationnaire sur des surfaces planes de milieux poreux. Dans le cas d'un environment stratifié thermiquement stable, des solutions de similitude existent seulement pour la convection stationnaire sur plaques planes.

MÖGLICHE ÄHNLICHKEITSLÖSUNGEN FÜR GRENZSCHICHTEN NAHE EBENEN PLATTEN BEI FREIER KONVEKTION IN PORÖSEN MEDIEN

Zusammenfassung—In dieser Arbeit werden die notwendigen und hinreichenden Bedingungen geprüft, unter welchen für Grenzschichten nahe ebenen Platten bei freier Konvektion in porösen Medien Ähnlichkeitslösungen existieren. Es wurde festgestellt, daß bei stationärer freier Konvektion Ähnlichkeitslösungen existieren, wenn die Temperaturdifferenz zwischen Wand und Umgebung bei vertikalen Platten in Potenz- oder Exponentengliederform und bei horizontalen Platten in Potenzgliederform variiert. Es existieren auch mehrere spezielle Lösungen für instationäre freie Konvektion an ebenen Platten in porösen Medien. Für eine thermisch stabil geschichtete Umgebung existieren Ähnlichkeitslösungen nur bei stationärer freier Konvektion an vertikalen Platten.

ВОЗМОЖНЫЕ АВТОМОДЕЛЬНЫЕ РЕШЕНИЯ ДЛЯ СВОБОДНОКОНВЕКТИВНЫХ ПОГРАНИЧНЫХ СЛОЕВ У ПЛОСКИХ ПЛАСТИН В ПОРИСТЫХ СРЕДАХ

Аннотация — Анализируются необходимые и достаточные условия для получения автомодельных решений для свободноконвективных пограничных слоев у плоских пластин в пористых средах. Найдено, что в случае стационарной свободной конвекции автомодельные решения для вертикальных пластин возможны, когда разность температур между стенкой и окружающей средой изменяется по степенному и экспоненциальному законам, а для горизонтальных, когда она изменяется по степенному закону. Кроме того существует несколько специфических решений в случае нестационарной свободной конвекции у пластин в пористой среде. Для устойчивой термически стратифицированной среды автомодельные решения существуют только для стационарной свободной конвекции у вертикальных пластин.